

Polar Review

Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.

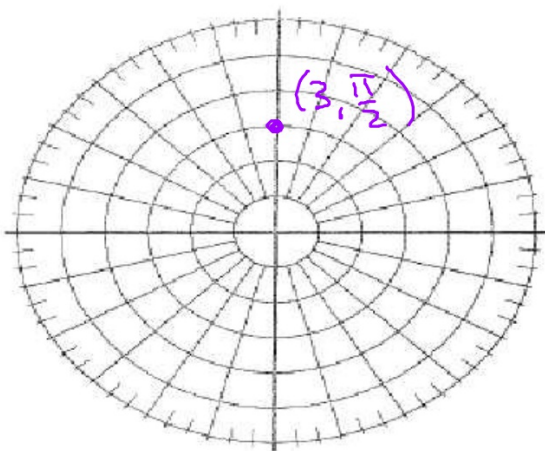
8) $\left(3, \frac{\pi}{2}\right)$

$$x = r \cos \theta \quad y = r \sin \theta$$

$$x = 3 \cos \frac{\pi}{2} \quad y = 3 \sin \frac{\pi}{2}$$

$$x = 0$$

$$y = 3$$



Plot the point whose polar coordinates are given. Then find the Cartesian coordinates of the point.

17) $\left(2\sqrt{2}, \frac{3\pi}{4}\right)$

$$x = 2\sqrt{2} \cos \frac{3\pi}{4}$$

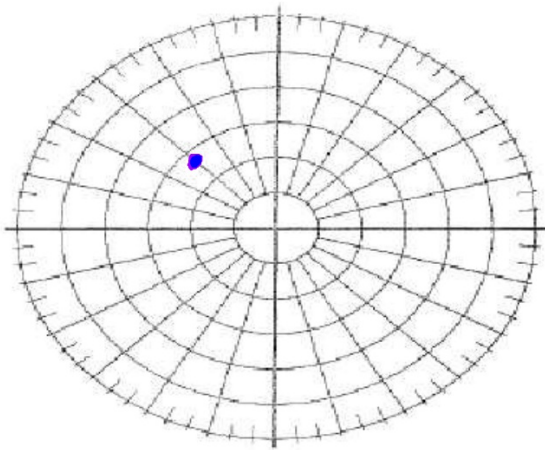
$$y = 2\sqrt{2} \sin \frac{3\pi}{4}$$

$$x = 2\sqrt{2} \left(-\frac{\sqrt{2}}{2}\right)$$

$$y = 2\sqrt{2} \left(\frac{\sqrt{2}}{2}\right)$$

$$x = -2$$

$$y = 2$$



The Cartesian coordinates of a point are given.
Plot the point and then find 4 polar representations of the curve

9) $(2\sqrt{3}, -2)$

$(4, -30^\circ)$

$(4, 330^\circ)$

$(-4, 150^\circ)$

$(-4, -210^\circ)$

$$r = \sqrt{(2\sqrt{3})^2 + (-2)^2} \quad \theta = \tan^{-1}\left(\frac{-2}{2\sqrt{3}}\right)$$

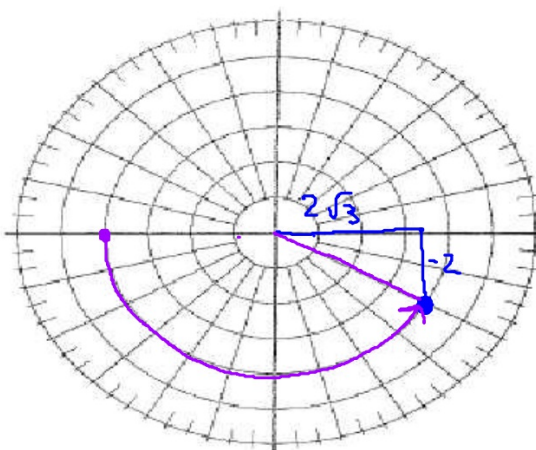
$$r = \sqrt{(4 \cdot 3) + 4}$$

$$r = \sqrt{16} = 4$$

$$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$$

$$\theta = \tan^{-1}\left(\frac{-1/2}{\sqrt{3}/2}\right)$$

$$\theta = -30^\circ$$



The Cartesian coordinates of a point are given.
Plot the point and then find 4 polar
representations of the curve

19) $(-1, -\sqrt{3})$

$(-2, 60^\circ)$

$(-2, -300^\circ)$

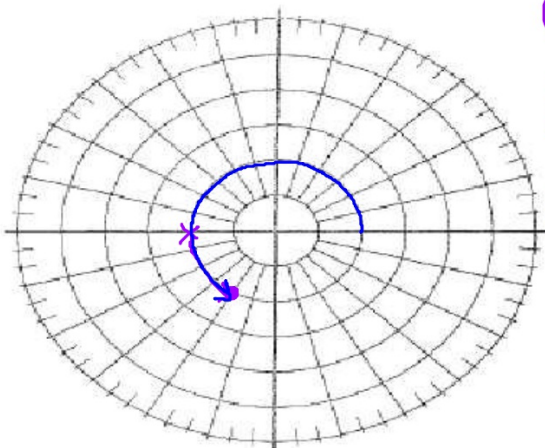
$(2, 240^\circ)$

$(2, -120^\circ)$

$$r = \sqrt{1^2 + \sqrt{3}^2} = \sqrt{1+3} = 2$$

$$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right)$$

$$\theta = 60^\circ$$



Identify the curve by finding a
Cartesian/Rectangular equation for the curve.

12. $r = 2$ *circle*

$$\sqrt{x^2 + y^2} = 2 \quad r = \sqrt{x^2 + y^2}$$

$$x^2 + y^2 = 4$$

Identify the curve by finding a
Cartesian/Rectangular equation for the curve.

$$\underline{r \cos \theta = 1}$$

$$x = 1$$

Vertical Line

Identify the curve by finding a Cartesian/Rectangular equation for the curve.

$$(r = 3 \sin \theta) \quad r$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$r = \sqrt{x^2 + y^2}$$

$$r^2 = 3r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$x^2 + y^2 = 3y$$

Circle

Identify the curve by finding a Cartesian/Rectangular equation for the curve.

$$r = \tan \theta \sec \theta$$

$$(\cancel{\cos \theta}) r = \left(\frac{\sin \theta}{\cos \theta} \right) \left(\frac{1}{\cancel{\cos \theta}} \right) (\cancel{\cos \theta})$$

$$r \cos \theta = \frac{\sin \theta}{\cos \theta}$$

$$(\cancel{\cos \theta}) x = \frac{\sin \theta}{\cancel{\cos \theta}} (\cancel{\cos \theta})$$

$$\rightarrow (x \cos \theta = \sin \theta) r$$

$$x(r \cos \theta) = r \sin \theta$$

$$x(x) = y$$

$$x^2 = y$$

Parabola

Find a polar equation for the curve represented by the given Cartesian equation.

16. $x = 3$

$$\frac{r \cos \theta}{\cos \theta} = \frac{3}{\cos \theta}$$

$$r = \frac{3}{\cos \theta}$$

$$r = 3 \sec \theta$$

Find a polar equation for the curve represented by the given Cartesian equation.

$$x^2 + y^2 = 9$$

$$r^2 = 9$$

Circle

Find a polar equation for the curve represented by the given Cartesian equation.

$$x = -y^2$$

$$r \cos \theta = - (r \sin \theta)^2$$

$$r \cos \theta = -r^2 \sin^2 \theta$$

Find a polar equation for the curve represented by the given Cartesian equation.

$$x + y = 9$$

$$r \cos \theta + r \sin \theta = 9$$

$$r (\cos \theta + \sin \theta) = 9$$

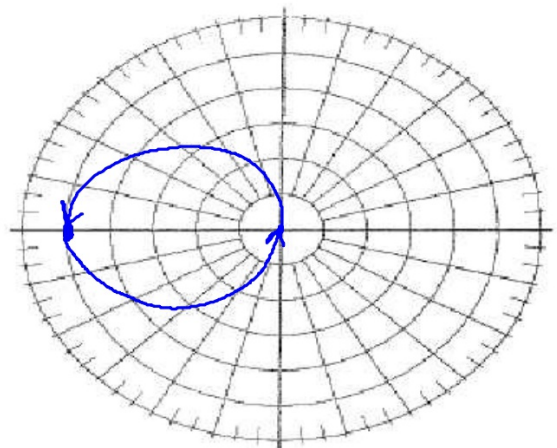
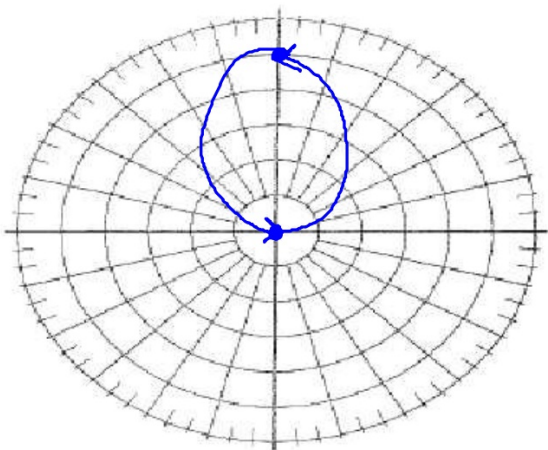
$$r = \frac{9}{\cos \theta + \sin \theta}$$

Sketch the curve with the given polar equation

Circle

$$r = 5 \sin \theta$$

$$r = -5 \cos \theta$$



Sketch the curve with the given polar equation

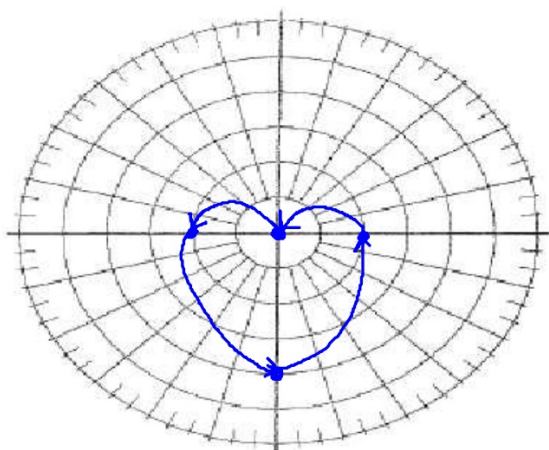
CARDIoid

$$r = 2 - 2\sin\theta$$

right/left

$$\theta = 0$$

$$r = 2$$

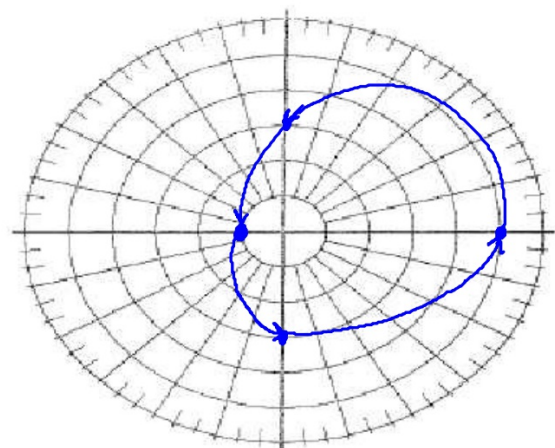


Dimpled
Limacon

$$r = -3 + 2\cos\theta$$

$$\theta = 0$$

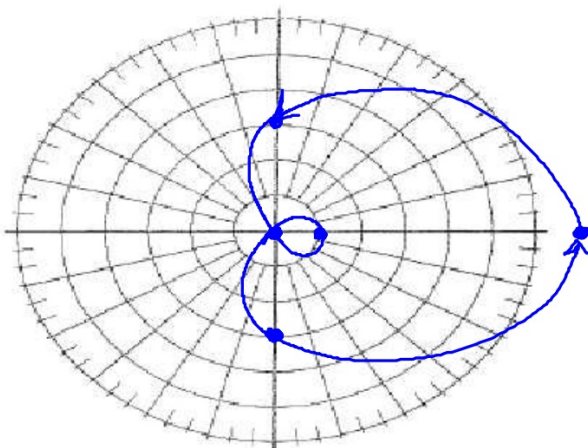
$$r = -3 + 2 = -1$$



Sketch the curve with the given polar equation

↓ Loop
 $r = 3 + 4\cos\theta$

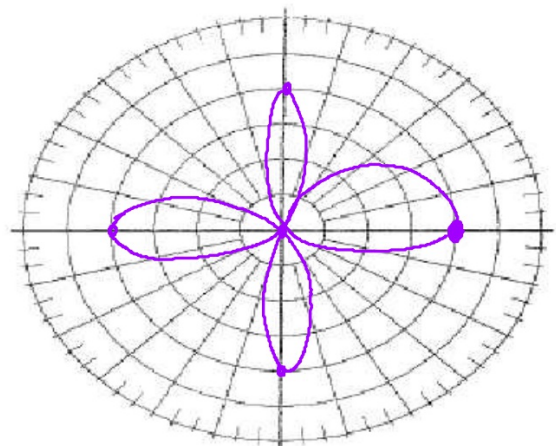
$\theta = 0$
 $r = 7$



Rose
 $r = 4\cos 2\theta$

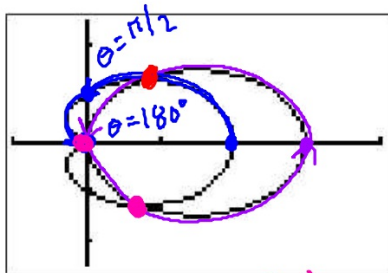
$\frac{360}{4} = 90^\circ$

$\theta = 0$
 $r = 4$



- Find where each of the curves is when $\theta = 0$.
- Find where each of the curves is when $\theta = \frac{\pi}{2}$
- Using your information from parts a and b identify the direction the curve is moving.
- Find when each curve is at the pole. e) Find where the 2 curves intersect.

26. the circle $r = 3\cos\theta$ and the cardioid $r = 1 + \cos\theta$



Pole: circle $\theta = 90^\circ$
cardioid $\theta = 180^\circ$

$$r = 3\cos\theta$$

$$a) \theta = 0 \quad r = 3$$

$$b) \theta = \frac{\pi}{2} \quad r = 3\cos\frac{\pi}{2} \\ r = 0$$

$$c) \theta = \pi/2$$

$$a) \theta = 0 \quad r = 1 + 1 = 2$$

$$b) \theta = \frac{\pi}{2} \quad r = 1 + 0 = 1$$

$$d) 0 = 1 + \cos\theta$$

$$\begin{array}{r} -1 \quad -1 \\ \hline -1 = \cos\theta \end{array}$$

$$\theta = 180^\circ$$

$$\begin{array}{r} 3\cos\theta = 1 + \cos\theta \\ -\cos\theta \quad -\cos\theta \\ \hline \end{array}$$

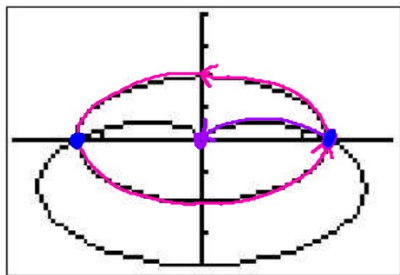
$$\frac{2\cos\theta}{2} = \frac{1}{2}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

- a) Find where each of the curves is when $\theta = 0$.
 b) Find where each of the curves is when $\theta = \frac{\pi}{2}$.
 c) Using your information from parts a and b identify the direction the curve is moving.
 d) Find when each curve is at the pole. e) Find where the 2 curves intersect.

27. the circle $r = 2$ and the cardioid $r = 2(1 - \sin\theta)$



$\theta = 0 \quad r = 2$
 $\theta = \frac{\pi}{2} \quad r = 2$
 never at pole

$\theta = 0 \quad r = 2(1 - 0) = 2$
 $\theta = \frac{\pi}{2} \quad r = 2(1 - 1) = 0$
 at pole $\theta = \frac{\pi}{2}$

$$\begin{array}{r} 0 = 2 - 2\sin\theta \\ -2 \quad -2 \\ \hline -2 = -2\sin\theta \\ 1 = \sin\theta \end{array}$$

$$2 = 2(1 - \sin\theta)$$

$$2 = 2 - 2\sin\theta$$

$$\begin{array}{r} 0 = -2\sin\theta \\ -2 \quad -2 \\ \hline \end{array}$$

$$0 = \sin\theta$$

$$\boxed{\theta = 0^\circ / 180^\circ}$$

	sin	<u>cos</u>
30	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
45	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
<u>60</u>	$\frac{\sqrt{3}}{2}$	<u>$\frac{1}{2}$</u>

